

1481 : Q10 :

$$\frac{dy}{dx} = -y^2 \cos^3 x$$

$$\Rightarrow \int \frac{1}{y^2} dy = \int \cos^3 x dx. \quad (*)$$

$$\cos^3 x = \cos^2 x \cos x \\ = (1 - \sin^2 x) \cos x$$

$$\begin{aligned} \therefore \int \cos^3 x dx &= \int (1 - \sin^2 x) \cos x dx \\ &= \int [\cos x - \sin^2 x \cos x] dx \\ &= \int \cos x dx - \int \sin^2 x \cos x dx \\ &= \sin x - \int u^2 du \quad \begin{matrix} u = \sin x \\ du = \cos x dx \end{matrix} \\ &= \sin x - \frac{u^3}{3} + C \\ &= \sin x - \frac{\sin^3 x}{3} + C \end{aligned}$$

$$(*) \Rightarrow -\left(\frac{1}{y}\right) = \sin x - \frac{\sin^3 x}{3} + C$$

$$\Rightarrow \frac{1}{y} = \sin x - \frac{\sin^3 x}{3} + C$$

$$y=2, x=\frac{\pi}{6} \Rightarrow \frac{1}{2} = \sin \frac{\pi}{6} - \frac{\sin^3 \frac{\pi}{6}}{3} + C$$

$$\Rightarrow C = \frac{1}{24}$$

$$\therefore \frac{1}{y} = \sin x - \frac{\sin^3 x}{3} + \frac{1}{24}$$

$$\Rightarrow y = \left[\sin x - \frac{\sin^3 x}{3} + \frac{1}{24} \right]^{-1}$$